LESSON PLAN STUDY ON HOW TO CHOOSE EXAMPLES AND PREPARE EXPLANATIONS FOR HYPERBOLA GRAPH IN GRADE 10

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This paper reports on a study that was conducted over a period of three weeks on 'how to plan a lesson to help learners to understand the meaning of the asymptote from the formula and graph of hyperbola'. The main focus of the study was on the meaning of the asymptote when the learners sketch a hyperbola graph as well as deliberate selection and use of examples by teachers. During the period of three weeks, the study was divided into three phases, namely: planning, Laboratory lesson1 by the first teacher and Laboratory lesson 2 by the second teacher. Even though it's not possible to conduct such lessons every day, but this study gave us a good practice on how lessons should be planned and conducted.

INTRODUCTION

In this paper we share what we did in a lesson plan study that was initiated by Wits Maths Connect Secondary (WMCS) Project. Therefore, we will briefly give a description of the project, the framework that we used, the rationale for the chosen focus which was the hyperbolic function, and then the three phases that constituted the three weeks of lesson planning respectively.

The project

WMCS is a project that is based at the University of the Witwatersrand, Johannesburg in the Wits School of Education. WMCS offers two Maths courses called Transition Mathematics 1 and 2 (TM1 and TM2), to help Maths educators to develop their knowledge in mathematics for teaching. I and my colleague who is writing this paper with me attended TM1 and now we are attending TM2. What we share in this paper is how we experienced one aspect of the professional development and that is the school based component called lesson plan study. To do the lesson plan study we had a framework that was developed by the project.

The framework

Firstly, the framework enables us to think about the object of learning, that is to say the capability that you want developed in learners by the end of the lesson. To achieve this, the framework brings a teachers attention to the deliberate selection and use of examples to explain a particular concept. The framework also allows us to think about the activities that learners will be engaged in (see appendix 1).

Functions

We identified the topic of the hyperbolic function as the one we would work with over the three weeks for two reasons. Firstly, in the official examinations paper one has more questions based on functions. Secondly functions are a tool used to solve various complex problems in Maths, Science and Technology environment. We saw it very important therefore for us to focus on this topic and see how best we can teach it in such a way that learners learn. Achieving this would then help learners improve their ability when approaching mathematical problems in general. We now move on to discuss the three phases, namely: the planning phase, laboratory lesson one and lastly laboratory lesson two.

PHASE 1: PLANNING

Participants

There were six Maths educators and three members from WMCS project that were participants during this phase. Two educators that were part of the team are Maths HOD's with more than ten years of experience teaching Maths at FET level and other educators are post level one educator from two different schools.

In the planning session we were guided by a Mathematics Teaching Framework (MTF) discussed above and shown in the appendix. The MTF lesson plan tool was used as guide for choosing examples, preparing explanations, and learner activity.

Rationale for choice of examples

In selecting the examples we looked for examples that would address learner misconceptions and learner difficulties in interpreting and drawing the hyperbola graphs during tests or exams. Learners pre-knowledge and misconceptions about hyperbola graph were put into consideration in the process of selecting and choosing examples. Annexture two shows the examples that were planned to overcome those misconceptions and they were mainly selected from Classroom Mathematics Grade 10 authored by Laridon et al (2011).

PHASE 2: LABORATORY LESSON 1 AND REFLECTION

This was a first lesson of a study and was presented by teacher 1 using the MTF planned in phase 1. I am a post level one educator who attended TM1course with WMCS and I am the one who was responsible to present the first lesson. The duration of a lesson was one hour.

2.1 Lesson presentation 1

<u>Participants</u>

Participants in the lesson were 20 Grade 10 Maths learners, observed by five educators and four WMCS project team members. Educators and Wits members were seated at the back of the classroom during the lesson and six of them were part of the planning stage of the study. Tables were arranged in a way that allows the learners to seat in pairs and the learners were also allowed to work in pairs.

As this was a revision lesson, there was no need to teach learners how to draw a hyperbola graph. The aim was to help the learners to understand the meaning and the effect of the asymptote to the hyperbola graph. The learners were given six equations as shown in figure 1 in annexure 2and they were asked to use table method to draw the graphs. After drawing graphs, learners were given cards with the same six equations and six graphs to match. This was a very interesting exercise to the learners, because learners had to look at their graphs and compare their graphs with the graphs given in the cards. Even the learners who did not get their graph correct at the beginning of a lesson, they managed to find the card matching correct. They used equations and different features of the graphs; see annexure 3.

After doing card matching exercise, learners were given another exercise to compare the first graph with other five graphs. They were looking at what are common features and what are different features between the graphs. During this exercise, the learners' comparisons were based on how the quadrants shifted. They did not talk about the asymptotes when looking at graphs. They only talked about vertical shifts. The only time when they introduced the word 'asymptote' is when they were trying to use equations to explain their observations about graphs. The learners could see the asymptote from the equation, but they only see the lines shifting from the graphs which they spoke about as quadrants shifting. .

The last exercise for the day was to sketch the hyperbola graphs without using table method. Even though initially this exercise was not planned to be the last, but due to the amount of time that was remaining, the educator was forced to make it to be the last. The activity in figure 2 was initially planned to be the last. The activity in figure 2 was planned to help the learners to observe what happens to the values of y as the values of x get very closed to an asymptote, get very small and get very big. This activity was also useful to help the learners to understand the meaning of the asymptote.

Graph sketching activity

The sketching exercise went very fast and proved to be the most interesting exercise to learners. Learners' participation and solutions displayed to have an understanding of the asymptote from the equation to the graph. However, when it came to the: $y = \frac{x}{2} + 1$. All learners produced a hyperbola graph for this equation. They did not notice that this is no longer a hyperbola graph but it's a straight line graph. It was particularly a useful part of the entire experience because it showed the importance of selecting examples. This marked the end of the lesson, learners left and we remained to reflect and discuss the lesson.

2.2 Reflection

In the reflection and discussion session we focused on what was the object of learning; what examples and explanations were offered. For examples additions were made for the next lesson and those changes from first lesson will appear under lesson 2.

The lesson that we all took from having carefully thought through the selection of examples and preparation of explanations in the planning is that good examples do not only help to expose learners misunderstanding, but also assist in helping learners to understand the meaning around the concept. The reflection and discussion made it possible to reveal learners' misconceptions/ misunderstanding and helped inform our planning for the next lesson that was going to be taught by a different teacher to a different group of grade 10 learners.

Phase 3: Laboratory lesson 2 and Reflection

Laboratory lesson 2 was a final stage of the study of the lesson preparation and execution. It was therefore the second lesson executed by teacher 2 after we re-planned the same lesson taught by the first teacher. I am a level one teacher who participated in TM1 course offered by WMCS in 2012. The lesson was taught for duration of 1 hour. In this phase, I would elaborate on this lesson referred to as laboratory lesson 2 and report on its reflection.

3.1 Lesson presentation 2

The participants were a new group of 22 Grade 10 learners (who were not involved in lesson 1) from my school, 5 WMCS members, 2 mathematics teachers from a neighbouring school as well as 2 mathematics colleagues. The members from WMCS and teachers observed the lesson by sitting at the back of the classroom (there were instances when they moved around as learners were engaged with class activities). Learners were working in pairs. Prior to the lesson, learners were givenhomework (same as in lesson 1) to draw the following functions: $1.\ y = \frac{2}{x}$; $2.\ y = \frac{-2}{x}$; $3.\ y = \frac{2}{x} + 3$ and $4.\ y = \frac{2}{x} - 3$. The lesson itself was segmented into the following points: (a) general form $y = q + \frac{a}{x}$, (b) asymptote and (c) the effect of a and q which i discuss below in that order.

(a) Discussion around $y = \frac{a}{x} + q$ and $y = q + \frac{a}{x}$

I began the lesson by asking learners to compare each other's homework. I then gave them the card matching activity as in Figure 1, except that we used $y = 3 - \frac{2}{x}$ for equation six instead of $y = -\frac{2}{x} - 3$. The six graphs for the matching activity were the same as in lesson 1. This activity was a follow up on learners' homework where they were expected to match an equation with a corresponding graph (see Table 1in the appendix). There were actually five equations because the sixth equation was a repetition of $y = -\frac{2}{x} + 3$ but written in a form of $y = 3 - \frac{2}{x}$. These two equations were included purposefully to explore if learners would be able to observe that they are the same. Hence, one of the graphs had no corresponding equation. When I asked learners what is the equation of the last graph (see Figure 3 below), they all shouted that it was obvious referring to $y = 3 - \frac{2}{x}$.

However, one learner noted that the asymptote of the graph is -3 and not 3, while others claimed that it was a computer error. They eventually discovered that $y = 3 - \frac{2}{x}$ was not a corresponding equation for graph in figure 3, and hence noticed that the two equations were the same. This was an interesting part of the activity because learners are so used to the general form of $y = \frac{a}{x} + q$ instead of $y = q + \frac{a}{x}$. After this activity I then moved on to a discussion of the notion of asymptote.

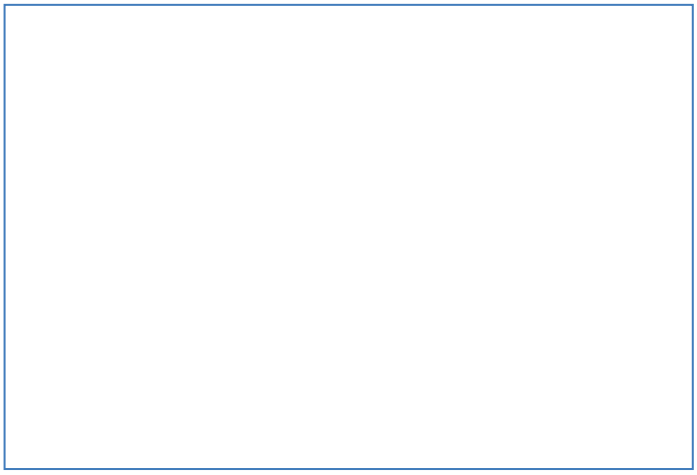

Figure 3: The hyperbola graph of $y = -\frac{2}{x} - 3$

(b) Discussion on asymptote

I then asked learners to focus on the graphs of equations 1 to 4 (these were functions given to learners for their homework) where I asked them to compare these graphs with respect to the things that changed and the things that are the same. According to Rowland (2008), variation allows learners to differentiate things. I first asked them to compare graph 1 and 3 as agreed in the re-planning session. Learners noted the movement of the graph by three units up, change of the range as well as the change of the horizontal asymptotes. I then engaged learners on the idea of the asymptote where I asked learners how many asymptotes a graph has. In most cases, learners do not recognise the vertical asymptote. They then realised that the vertical asymptote and the position of the graph stayed the same. I highlighted the issue of the position of the graph that we look at it with respect to the asymptotes, instead of shifting quadrants.

One learner noted that the line of symmetry has changed in graph 3 and it is $y = \frac{2}{x} + 3$. I did not expect this contribution especially because the learner gave the equation of symmetry. I recognised the learner by asking the class to applaud for her. I continued to ask learners to compare graphs 1 and 4 as well as graph 1 and 2 in terms of what changes and what stays the same.

(c) The effect of "a" and "q"

In the next stage of the lesson we discussed the effect of a and q on the graph of $y = \frac{a}{x} + q$.

However, learners were the ones to say that 'a' determines the position where the graph appears. They further said that q is the asymptote and it determines the movement of the graph. I then gave learners three more equations to draw rough sketch on the white board given to them (7.$y = \frac{4}{x} + 5$, 8. $y = 2 - \frac{4}{x}$ and 9. $y = \frac{x}{2} + 3$). Learners successfully sketched with the exception of graph number 9. Here only one learner recognised that it was a linear, while the rest drew a hyperbola. This was the last activity for the lesson. Learners were dismissed and we sat for a reflection and discussion session.

3.2 Reflection

WMCS members and mathematics teachers reflected on the lesson as soon as learners left. It was noted that the learners learnt what we expected them to learn. This was clearly demonstrated from the last activity. Again, the sequence of comparing graphs worked very well as compared to lesson 1.

We also reflected on the fact that it was a good idea to use six equations and graphs for card matching activity instead of five equations and graphs that we agreed upon during the re-planning session. This gave learners a platform to come up with the equation of the graph..

Phase3: *Laboratory lesson 2 and Reflection*

Laboratory lesson 2 was a final stage of the study of the lesson preparation and execution. Therefore, this was the second lesson executed by myself after we re-planned the same lesson taught by the first teacher. In this phase, I would elaborate on this lesson referred to as laboratory lesson 2 and report on its reflection.

Lesson presentation 2

The participants were a new group of 22 Grade 10 learners from my school, 5 Wits Maths Connect (WMC) officials, 2 mathematics teachers from a neighbouring school as well as 2 mathematics colleagues. The officials from WMC and teachers observed the lesson by sitting at the back of the classroom (there were instances when they moved around as learners were engaged with class activity). Learners were working in pairs. Prior to the lesson, learners were given a homework to draw the following functions: 1. $y = \frac{2}{x}$; 2. $y = \frac{-2}{x}$; 3. $y = \frac{2}{x} + 3$ and 4. $y = \frac{2}{x} - 3$.

I began the lesson by asking learners to compare each other's homework. I then gave them the card matching activity, which consisted of six equations and six graphs. This activity was a follow up on learners' homework where they were expected to match an equation with a corresponding graph (see Table 1in the appendix). There were actually five equations because the sixth equation was a repetition of $y = -\frac{2}{x} + 3$ but written in a form of $y = 3 - \frac{2}{x}$. These two equations were included purposefully to explore if learners would be able to observe that they are the same. Hence, one of the graphs had no corresponding equation.

When I asked learners what is the equation of the last graph, they all shouted that it was obvious referring to $y = 3 - \frac{2}{x}$. However, one learner noted that the asymptote of the graph is -3 and not 3, while others claimed that it was a computer error. They eventually discovered that $y = 3 - \frac{2}{x}$ was not a corresponding graph, hence noticed that the two equations were the same. This was an interesting part of the activity because learners are so used to the general form of $y = \frac{a}{x} + q$ instead of $y = q + \frac{a}{x}$.

I then asked learners to focus on the graphs of equations 1 to 4 (these were functions given to learners for their homework) where I asked them to compare these graphs with respect to the things that changed and the things that are the same. I first asked them to compare graph 1 and 3 as agreed in the re-planning session. Learners noted the movement of the graph by three units up, change of the range as well as the change of the horizontal asymptotes. I then build up on the idea of the asymptote where I asked learners how many asymptotes a graph has. In most cases, learners do not recognise the vertical asymptote. They then realised that the vertical asymptote and the position of the graph stayed the same. I highlighted the issue of the position of the graph that we look at it with respect to the asymptotes. One learner noted that the line of symmetry has changed in graph 3 and it is y=*x* + 3. I did not expect this contribution especially because the learner gave the equation of symmetry. I recognised the learner by asking the class to applaud for her. I continued to ask learners to compare graphs 1 and 4 as well as graph 1 and 2 in terms of what changes and what stays the same.

While comparing graph 1 and 2, the position of the graph seemed to be the main thing that changed. As agreed in the re-planning session that I must refrain from using quadrant, I labelled the Cartesian plane as region1 to region 4 instead of quadrant 1 to quadrant 4. Hence, I explained to learners why the graph was appearing in region 1 and region 3 for graph 1 as well as region 2 and 4 for graph 2. This was done by re-writing the equations as $xy = 2$ and $xy = -2$. I further elaborated that the product of xy can only be positive 2 in region 1 and 3. Again, the product of xy is only negative 2 in region 2 and 4. Majority of learners nodded their head to confirm that it make sense. One of the observers noted that one learner said, “Final someone is making sense and that explanation was needed”. However, it was noted by another observer that this learner wanted to extend the same idea of determining the position of equations 3 and 4 but got stacked. I then concluded this stage of the lesson by asking learners the name of the graph we dealt with as well as its general form.

In the next stage of the lesson I built up on the idea of the general form of the hyperbola in terms of the effect of “a” and “q”. However, learners were the ones to say that ‘a’ determines the position where the graph appears. They further said that q is the asymptote and it determines the movement of the graph.

I then gave learners three more equations to draw rough sketch on the white board given to them ($7. y = \frac{4}{x} + 5$, $8. y = 2 - \frac{4}{x}$ and $9. y = \frac{x}{2} + 3$). I asked them to put up their boards so that everyone would see their sketch. Furthermore, they were asked to draw each sketch on the chalkboard. Although majority of learners got the sketch of equation 7 and 8 correctly, but equation 9 seemed to be a challenge because they drew a hyperbola. However, one learner noticed that the equation was linear as she said, "*x* appears in the numerator not denominator". This indicated that learners were used to the general form but did not put the attention on the position of *x*. I then concluded the lesson by asking learners to define an asymptote.

Reflection

WMC and mathematics teachers conducted reflection immediately after the lesson. It was noted that the learners learnt what we expected them to learn. This was clearly demonstrated when they were sketching graphs on the chalkboard. Therefore, the lesson went well especially because learners contributed more than what we expected. Again, the sequence of comparing graphs worked very well as compared to lesson 1. It was noted that the position of the graph came up clearly, because teacher 2 explained it with respect to the asymptotes. However, it was easy for learners to grasp the explanation of graph position when 'q' was not included but this explanation was not extended for equations with 'q'. It was thus concluded that in future the lesson should incorporate example with the value of 'q' when explaining why the position of the graph appears in certain quadrants. It was also noted that introducing regions instead of quadrants would create a misconception especially when learners progress to the next grade and are taught by someone else.

We also reflected on the fact that it was a good idea to use six equations and graphs for card matching activity instead of five equations and graphs that we agreed upon during the re-planning session. This gave learners a platform to come up with the equation of the graph. Again, it allowed them to work with equations in a form of $y = \frac{a}{x} + q$ and $y = q + \frac{a}{x}$. However, it was noted that some learners said '3' is a slope and $\frac{-3}{x}$ is an asymptote. This raised a question of what is learners' explanation of swapping $\frac{a}{x}$ and 'q'.

CONCLUSION

This was a very good exercise for our development as teachers. Inputs from different people were very useful in terms of influencing and shaping our thinking around the deliberate selection and use of examples and careful preparation of explanations. The planning phase gave us confidence to present the lessons and the reflection sessions indicated that more changes needed to be done in the same lesson. This indicated to us that there will never be a time where you come to a point of a good lesson that needs no improvement; there will always be space for comment and more improvement. Therefore, we learnt that the process of lesson planning is continuous.

This means that every time one plans a lesson you must expect changes because we are teaching learners with different thinking abilities. It would be great if such studies are implemented in other schools. Henceforth, we look forward to see these kinds of lessons taking place more often during 2014. This was a good exercise for teachers to develop and also for the learners to learn better.

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Annexure 1

Use table method to draw the graphs of the following functions:

1. $y = \frac{2}{x}$
2. $y = \frac{-2}{x}$
3. $y = \frac{2}{x} + 3$
4. $y = \frac{2}{x} - 3$
5. $y = \frac{-2}{x} + 3$
6. $y = \frac{-2}{x} - 3$

Use the equation: $y = \frac{2}{x}$ to complete the following tables:

(a)

x	0.01	0.001	0.0001	0.00001	0.000001
y					

(b)

x	- 0.01	- 0.001	- 0.0001	- 0.00001	- 0.000001
y					

(c)

x	100	1000	10 000	100 000
y				

(d)

X	- 100	- 1000	- 10 000	- 100 000
y				